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APPLYING PIAGET'S THEORY OF COGNITIVE DEVELOPMENT TO MATHEMATICS INSTRUCTION

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Abstract

Jean Piaget's cognitive development theory discusses how an individual progresses through the learning process in stages. This paper focuses on Jean Piaget's developmental stages and how to apply this theory to the learning of mathematics. Each stage has been described and characterized, highlighting appropriate mathematical techniques that help to lay a solid foundation for learning of mathematics in future. General implications of the knowledge of stages of development have been incorporated for instructions in mathematics.

Keywords **-** *Cognitive Development, Infinite Sets, Jean Piaget's Theory, Mathematics Instruction*

Introduction

"The principle goal of education in the schools should be creating men and women who are capable of doing new things, not simply repeating what other generations have done."

- *Jean Piaget*

Jean Piaget was a forefather of modern child development theory. Surprisingly, this educational academician began his career in the natural sciences. A quick turn to psychoanalysis brought an interest in human learning and knowledge acquisition.

Jean Piaget's work on children's cognitive development, specifically with quantitative concepts, has garnered much attention within the field of education. Piaget explored children's cognitive development to study his primary interest in genetic epistemology. Upon completion of his doctorate, he became intrigued with the processes by which children achieved their answers. He used conversation as a means to probe children's thinking based on experimental procedures used in psychiatric questioning.

Jean Piaget's cognitive development theory has strongly influenced the way how we view the learning process of individuals and the process that people go through while constructing their own knowledge. This is especially applicable in the mathematics discipline. Mathematics concepts build off of one another and intertwine and Jean Piaget's theory considers the steps through which process occurs.

Piaget's stages of cognitive development encompass growth from birth to adolescence. He identified four stages i.e. sensori-motor stage (birth to 2 years), the pre-operational stage (approximately 2 to 7), the concrete operational stage (7 to early adolescence) and the formal operational stage (adolescence). Educators can use the developmental theories behind each stage to create age-graded strategies for teaching mathematics.

Stages of Cognitive Development

As discussed above, Piaget has identified four primary stages of development i.e. sensori-motor, pre-operational, concrete operational and formal operational stage.

Sensori-motor Stage

In the sensori-motor stage, an infant's mental and cognitive attributes develop from birth until the appearance of language. At this point, children will be able to compare themselves to their parents or compare sizes of toys. Since they are unable to see different points of view, they will be limited to how situations or objects appear to them and will be restricted to concrete experiences and objects.

This stage is characterized by the progressive acquisition of object permanence in which the child becomes able to find objects after they have been displaced, even if the objects have been taken out of his field of vision. For example, Piaget's experiments at this stage include hiding an object under a pillow to see if the baby finds that object.

An additional characteristic of children at this stage is their ability to link numbers to objects (Piaget, 1977) (e.g. one dog, two cats, three pigs, four hippos). To develop the mathematical capability of a child at this stage, the child's ability might be enhanced if he is provided an ample opportunity to act on the environment in unrestricted (but safe) ways in order to start building concepts (Martin, 2000). Evidence suggests that children at the sensori-motor stage have some understanding of the concepts of numbers and counting (Fuson, 1988). Educators of children at this stage of development should lay a solid mathematical foundation by providing activities that incorporate counting and thus, enhance children's conceptual development of numbers. For example, teachers and parents can help children count their fingers, toys and candies. Questions such as "Who has more?" or "Are there enough?" could be a part of the daily lives of children as young as two or three years of age.

Another activity that could enhance the mathematical development of children at this stage connects mathematics and literature. There is a plethora of children's books that embed mathematical content. A recommendation would be that these books include pictorial illustrations. Because children at this stage can link numbers to objects, learners can benefit from seeing pictures of objects and their respective numbers simultaneously. Along with the mathematical benefits, children's books can contribute to the development of their reading skills and comprehension.

Pre-operational Stage

The characteristics of this stage include an increase in language ability (with over-generalizations), symbolic thought, egocentric perspective and limited logic. In this second stage, children should engage with problem-solving tasks that incorporate available materials such as blocks, sand and water. While the child is working with a problem, the teacher should elicit conversation from the child. The verbalization of the child as well as his actions on the materials gives a basis that permits the teacher to infer the mechanisms of the child's thought processes.

There is a lack of logic associated with this stage of development. Rational thought makes little appearance. The child links together unrelated events, sees objects as possessing life, does not understand point-of-view and cannot reverse operations. For example, a child at this stage who understands that adding four to five yields nine cannot yet perform the reverse operation of taking four from nine.

Children's perceptions at this stage of development are generally restricted to one aspect or dimension of an object at the expense of the other aspects. For example, Piaget tested the concept of conservation by pouring the same amount of liquid into two similar containers. When the liquid from one container is poured into a third wider container, the level gets lowered and the child thinks that there is less liquid in the third container. Thus, the child is using one dimension i.e. height as the basis for his judgment of another dimension i.e. volume.

A teacher should include concepts that easily translate into teaching strategies. For example, using the idea that the child might understand the connection between an object and the symbol that it represents. He should set-up a hands-on-number lesson in which groups of toys or other objects represent numbers such as five toy cars, three apples or seven pieces of chalk.

A teacher should write a lesson plan detailing each step and its relation to Piaget's theory and should note the specific stage (i.e. pre-operational) and theory idea (e.g. make-believe/fantasy, representation). He should design a specific learning goal or object such as students learning to count to 10 by themselves or child recognizing written numerals and make a bulleted list of materials and a numbered list of steps.

Teaching students at this stage of development should employ effective questioning about characterizing objects. For example, when students investigate geometric shapes, a teacher could ask students to group the shapes according to similar characteristics. Questions following the investigation could include "How did you decide where each object belonged? Are there other ways to group these together?" Engaging in such discussions or interactions with the children may engender the children's discovery of the variety of ways to group objects. Thus, helping the children think about the quantities in novel ways (Thompson, 1990).

Concrete Operational Stage

The third stage is characterized by remarkable cognitive growth, when children's development of language and acquisition of basic skills accelerate dramatically. Children at this stage utilize their senses in order to *know*. They can now consider two or three dimensions simultaneously instead of successively. For example, in the liquids experiment, if the child notices the lowered level of the liquid, he also notices the dish is wider, seeing both dimensions at the same time. Additionally, seriation and classification are the two logical operations that develop during this stage (Piaget, 1977) and both are essential for understanding number concepts. Seriation is the ability to order objects according to increasing or decreasing length, weight or volume. On the other hand, classification involves grouping objects on the basis of a common characteristics.

At this stage, the child can accurately compare the two different containers. The child understands that pouring the liquid into a different container does not change the amount of liquid. Also, the child understands that containers that have the same amount of liquid but different dimensions will appear to have differing amounts of liquid because they have different levels of liquid. The child takes comparing these situations to a higher level and accurately compares these two containers of liquid.

According to Burns & Silbey (2000), "Hands-on experiences and multiple ways of representing a mathematical solution can be the ways of fostering the development of this cognitive stage". The importance of hands-on activities cannot be over-emphasized at this stage. These activities provide students an avenue to make abstract ideas concrete, allowing them to get their hands on mathematical ideas and concepts as useful tools for solving problems. Because concrete experiences are needed, teachers might use manipulatives with their students to explore concepts such as place value and arithmetical operations. Existing manipulative materials include pattern blocks, cuisenaire rods, algebra tiles, algebra cubes, geo-boards, tangrams, counters, dice and spinners. However, teachers are not limited to commercial materials. They can also use convenient materials in activities such as paper folding and cutting. As students use the materials, they acquire experiences that help lay the foundation for more advanced mathematical thinking. Furthermore, student's use of materials helps to build their mathematical confidence by giving them a way to test and confirm their reasoning.

One of the important challenges in mathematics teaching is to help students make connections between the mathematics concepts and the activities. Children may not automatically make connections between the work they do with manipulative materials and the corresponding abstract mathematics: "Children tend to think that the manipulations they do with models are one method for finding a solution and pencil-and-paper math is entirely separate" (Burns & Silbey, 2000). For example, it may be difficult for children to conceptualize how a four by six inch rectangle built with wooden tiles relates to four multiplied by six or four groups of six. Teachers could help students make connections by showing how the rectangles can be separated into four rows of six tiles each and by demonstrating how the rectangle is another representation of four groups of six.

Providing various mathematical representations acknowledges the uniqueness of students and provides multiple paths for making ideas meaningful. Engendering opportunities for students to present mathematical solutions in multiple ways (e.g. symbols, graphs, tables and words) is one tool for cognitive development at this stage. Eggen and Kauchak (2000) noted that while a specific way of representing an idea is meaningful to some students, a different representation might be more meaningful to others.

Formal Operational Stage

The child at this stage is capable of forming hypotheses and deducing possible consequences, allowing the child to construct his own mathematics. Furthermore, the child typically begins to develop abstract thought patterns where reasoning is executed using pure symbols without the necessity of perceptive data. For example, the formal operational learner can solve $x + 2x = 9$ without having to refer to a concrete situation presented by the teacher such as, "Tony ate a certain number of candies. His sister ate twice as many. Together they ate nine. How many did Tony eat?" Reasoning skills within this stage refer to the mental process involved in the generalizing and evaluating of logical arguments (Anderson, 1990) and include clarification, inference, evaluation, and application.

Clarification. Clarification requires students to identify and analyze elements of a problem, allowing them to decipher the information needed in solving a problem. By encouraging students to extract relevant information from a problem statement, teachers can help students enhance their mathematical understanding.

Inference. Students at this stage are developmentally ready to make inductive and deductive inferences in mathematics. Deductive inferences involve reasoning from general concepts to specific instances. On the other hand, inductive inferences are based on extracting similarities and differences among specific objects and events and arriving at generalizations.

Evaluation. Evaluation involves using criteria to judge the adequacy of a problem solution. For example, the student can follow a pre-determined rubric to judge the correctness of his solution to a problem. Evaluation leads to formulating hypotheses about future events, assuming one's problem solving is correct thus far.

Application. Application involves students connecting mathematical concepts to real-life situations. For example, the student could apply his knowledge of rational equations to the following situation: "You can clean your house in 4 hours. Your sister can clean it in 6 hours. How long will it take you to clean the house, working together?"

At this stage, the child further understands how to compare different mathematical situations. At this point in children's development, since they are not limited to the concrete world, they can compare fractions, the possibilities of different events and infinite sets. While infinity can cause an extended period of disequilibrium, the concept is beginning to be further understood. Having students examine infinite sets provides a situation in which we force the child to experience disequilibrium, in order for them to generate a deeper understanding of infinity. When comparing infinite sets, we use the idea of one-to-one correspondence to determine if two sets have the same number of elements. For example, when we compare two line segments that have different lengths, we can apply the idea of one-to-one correspondence to conclude they have the same number of points.

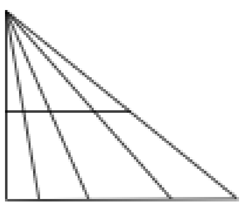

Figure 1. Two line segments having different lengths, both have the same number of infinite

points because there is an one-to-one correspondence between the points on the lines.

Figure 1 shows that we can match up every point on one line to a unique point on the second line. Further, each point on the second segment corresponds to a point on the first segment. So, even though one line may be contained in the second line. Since there are an infinite number of points on one line, there are equally many points on the second line segment.

Another example arises if we compare {1, 2, 3...} and {1, 1⁄2, 1/3...}. We can use one-to-one correspondence to see that these sets do in fact have the same number of elements. The unbounded set {1, 2, 3...} clearly has an infinite number of elements but it may be less clear when examining the second set, which is bounded, since many people's intuition of infinity includes the idea of unbounded. But if we use a one- to-one relationship, we can match 1 and 1, 2 and 1⁄2 and so on. Thus, the sets are the same size.

If we look at the idea of comparing infinite sets a little bit further, we arrive at the question about whether two infinite sets can possibly have different sizes. To examine this question, we compare the real numbers between 0 and 1 to the natural numbers, leading to the following proof.

Proof

Why is the set (0, 1) = {x: 0<x<1} not the same size as {1, 2, 3...}?

If they were the same size we would be able to have a 1-1 correspondence between the two sets. Something would be matched with 1 - lets call it x_1, something with 2 - call it x_2, something with 3 – call it x_3, and so on. Every number would be matched with a different integer, so the interval of real numbers between 0 and 1 would be listed x_1, x_2, x_3...

Claim: The list has a missing number.

Think of writing each number in expanded decimal form. We can then think of them going on forever, 1⁄2 = 0.50000.... and 1/3= 0.33333..... We note that some care is necessary though since 0.499... = 0.500....

So, let $x_1 = 0.x_{11}x_{12}x_{13}x_{14}...$ $x_2 = 0.x_{21}x_{22}x_{23}x_{24}...$ $x_3 = 0.x_{31}x_{32}x_{33}x_{34}...$ $x_4 = 0.x_{41}x_{42}x_{43}x_{44}...$

The first subscript indicates which number in the list to refer to and the second indicates which decimal place.

Now think about the number $y = 0.y_1y_2y_3y_4...$where $y_1 \neq x_{11}, y_2 \neq x_{22}, y_3 \neq x_{33}$ ...andsoingenerally$_n \neq x_{nn}$forn=1,2, 3...

Then y is between 0 and 1 but is not in the list because it differs from each number in the list at some point in its expansion. So, no list contains all the numbers from 0 to 1. Therefore, there is not a 1-1 correspondence and there must be more numbers from 0 to 1 than in the set of natural numbers. Thus, we are able to conclude that not all infinite sets are the same size.

This makes comparing infinite sets more interesting because when we conclude two sets have the same number of elements; it is not because they are both infinite but because they truly have the same number of elements. This example shows why there can be an extended period of disequilibrium even through the basic idea is an extension of an idea that children start to understand at early ages.

Overall Jean Piaget's Developmental theory allows us to see how a concept develops and outlines how a child progresses through this developmental process of learning. Examining Piaget's theory begs the question of whether we are pushing students in our educational system through this developmental process too quickly and asking them to learn concepts before they are capable of fully understanding these ideas. This leads to the importance of determining where a child is before presenting new information to fully address whether a child is capable of understanding the new material based on their current developmental stage. So, understanding how a child moves through this developmental process can enhance our understanding of how children learn and therefore increase the chances of understanding new complex ideas.

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